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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 7 2 3 0 4 5 4 4 0 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2025**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (PQ/SW) 342947/2

1 **(a)** Sketch the graph of $y = |3x - 2a|$, where a is a positive constant. [1]

(b) Hence or otherwise solve the inequality $|3x - 2a| < x + 5a$. [3]

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2 Solve the equation $2\ln(2x+3) - \ln(2x+5) = \ln(3x)$. [4]

3 Find the exact value of $\int_{\frac{1}{5}\pi}^{\frac{1}{4}\pi} 3\cos^2 5x \, dx$. [4]

4 **(a)** It is given that $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$.

Show that $(z_1 z_2)^* = z_1^* z_2^*$. [3]

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(b) $z = 3e^{\frac{1}{4}\pi i}$ is a root of the equation $z^2 + bz + c = 0$, where b and c are real.

State the other root and hence find the values of b and c. [3]

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5 The equation of a curve is $xy + y^2 e^{-x} = 4$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y^2 - ye^x}{xe^x + 2y}$. [4]

(b) Find the gradients of the tangents to the curve when $x = 0$. [2]

6 Find the complex numbers z for which $\frac{z+4}{z+4\mathrm{i}}$ is real and $|z| = \sqrt{10}$. Give your answers in the form $z = x + \mathrm{i}y$, where x and y are real. [6]

7 Let $f(x) = \dfrac{3a - 5x}{(3a + 2x)(2a - x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]

8 **(a)** Prove the identity $\cot^2\theta - \tan^2\theta \equiv 4\cot 2\theta \operatorname{cosec} 2\theta$. [4]

(b) Hence solve the equation $\cot^2 x - \tan^2 x = 5\sec 2x$ for $0° < x < 90°$. [4]

9 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j}, \quad \overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}.$$

The line l passes through B and C.

(a) Find a vector equation for l. [2]

(b) The point P is the foot of the perpendicular from A to l.

Find the position vector of P. [4]

(c) The point D is the reflection of A in l.

Find the position vector of D. [2]

10 The variables x and y satisfy the differential equation

$$\sin 4y \frac{dy}{dx} = x \sin 2y \sin 3x.$$

It is given that $y = \frac{1}{12}\pi$ when $x = \frac{1}{2}\pi$.

(a) Solve the differential equation, obtaining a relation between x and y. [8]

(b) Given that $0 < y < \frac{1}{2}\pi$, find the values of y when $x = 0$. [2]

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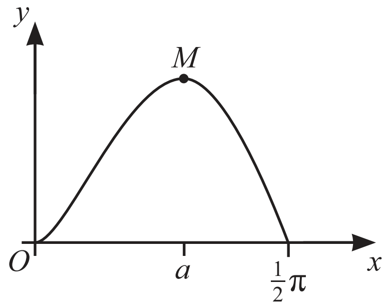

The diagram shows the curve $y = \sqrt{x}\sin 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve has a maximum point at M, where $x = a$.

(a) Show that $\tan 2a = -4a$ [4]

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(b) Show by calculation that $0.9 < a < 0.95$. [2]

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(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\left(\pi - \tan^{-1}(4x_n)\right)$$

converges, then it converges to a. [2]

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(d) Use the iterative formula in part **(c)** to calculate a correct to 4 decimal places. Give the result of each iteration to 6 decimal places. [3]

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Additional page

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